Learning to optimize computational resources:
Frugal training with generalization guarantees

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Algorithm configuration

Algorithms often have **tunable parameters**
- Impact resource consumption such as runtime, memory usage, …
- Hand-tuning is time-consuming and tedious

This paper: theoretical guarantees for algorithm configuration via ML

Learning-based configuration procedure
Input: Set of "typical" problem instances drawn from distribution Γ
E.g., integer programs (IPs) an airline solves day to day
Output: Parameter setting with low expected resource consumption
E.g., low expected runtime, memory usage, …

Goal: Procedure itself should have low resource consumption

Notation and example

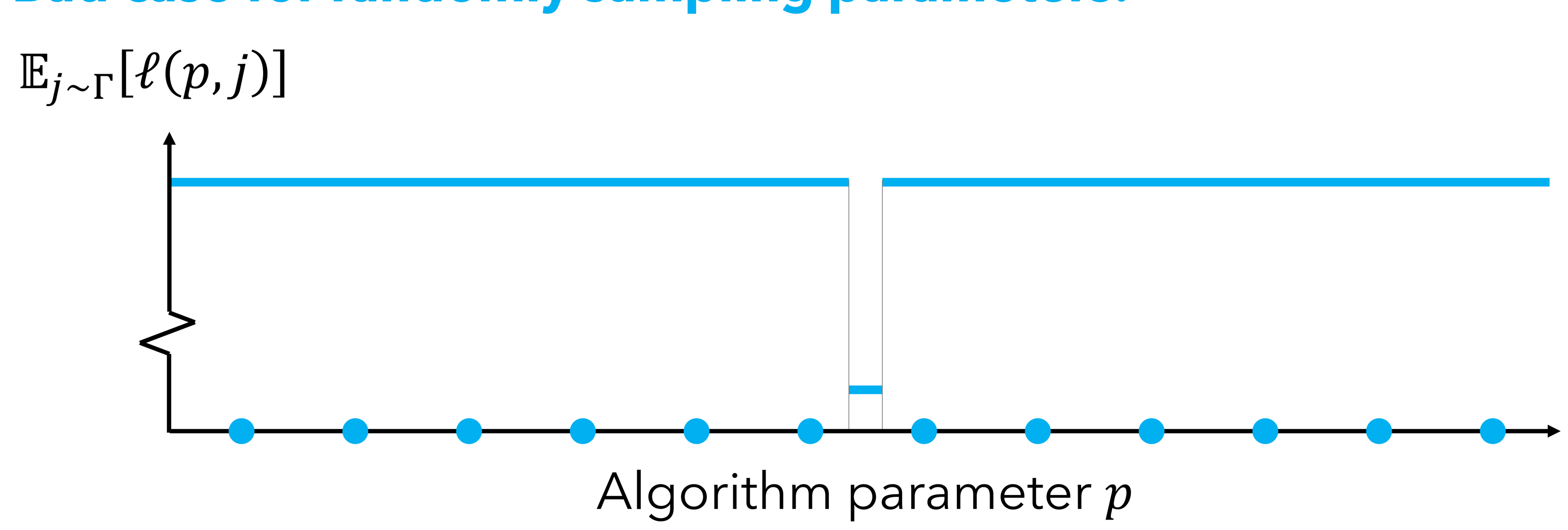
$\ell(p, j)$: Resources required to solve instance j using params $p \in \mathbb{R}^d$

Example: j = integer program and p = CPLEX parameter setting
$\ell(p, j)$ = size of branch-and-bound tree CPLEX builds

Prior research

Kleinberg et al. '17, '19 and Weisz et al., '18, '19:
Focus on finite parameter spaces
Can be used on infinite parameter space:
- Sample $\Omega\left(\frac{1}{\gamma}\right)$ configurations; run algorithm over finite set
- Output configuration is in top γ-quantile

Bad case for randomly sampling parameters:



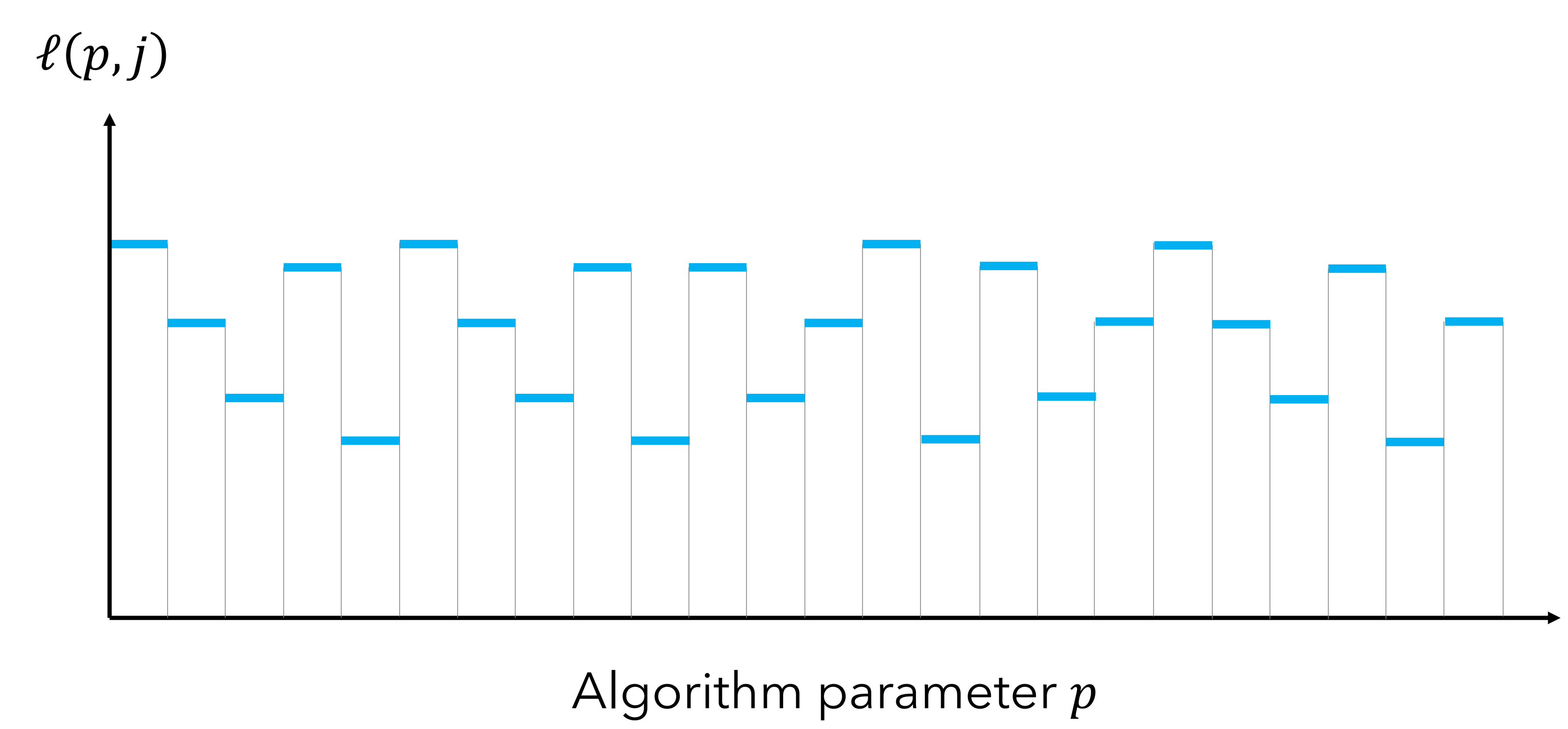
These worst-case examples do exist
E.g., in integer programming [Balcan, Dick, Sandholm, **V.** '18]

Our contributions

Algorithm that finds **finite** set of good params from within **infinite** set
- Set contains **nearly-optimal** parameter with high probability
- Can be used as input to algorithm for finite parameter spaces
  [Kleinberg et al. '17, '19; Weisz et al., '18, '19]

Useful (and requisite) structure

We often observe that $\ell(\cdot, j)$ is **piecewise-constant**
E.g., in integer programming [Balcan, Dick, Sandholm, **V.** '18]



Our algorithm

$\text{OPT} = \min_p \mathbb{E}_{j\sim\Gamma}[\ell(p,j)]$
(Actually compete with nuanced notion of OPT, like prior research
[Kleinberg et al. '17, '19; Weisz et al., '18, '19])

Maintains **upper confidence bound** (UCB) on OPT, initially set to ∞

On each round t, draws set S_t from Γ

Computes **partition** of parameters into regions where within each:
For each instance in S_t, the loss ℓ, capped at 2^t, is **constant**
Implementation guidance in prior research
[e.g., Balcan, Dick, Sandholm, **V.** '18]

On each region of partition, if enough instances have **loss less than 2^t**
Chooses arbitrary parameter from region and deems it "good"

Once cap 2^t has grown sufficiently large compared to UCB on **OPT**:
Algorithm returns set of "good" parameters

Guarantees

Theorem (informal):
1. WHP, exists "good" param in output that's within **$1 + \epsilon$ of optimal**
2. Algorithm terminates after $\tilde{O}\left(\ln\left(\sqrt[4]{1+\epsilon} \cdot OPT\right)\right)$ rounds
3. On final round, let P be the size of partition algorithm computes
   Number of "good" parameters is $\tilde{O}\left(P \cdot \ln\left(\sqrt[4]{1+\epsilon} \cdot OPT\right)\right)$
4. $|S_t|$ is polynomial in 2^t (linear in OPT), $\ln P$, d, and $\frac{1}{\epsilon}$

In bad case for random sampling, algorithm terminates in **$\tilde{O}(1)$ rounds**