Learned index structures

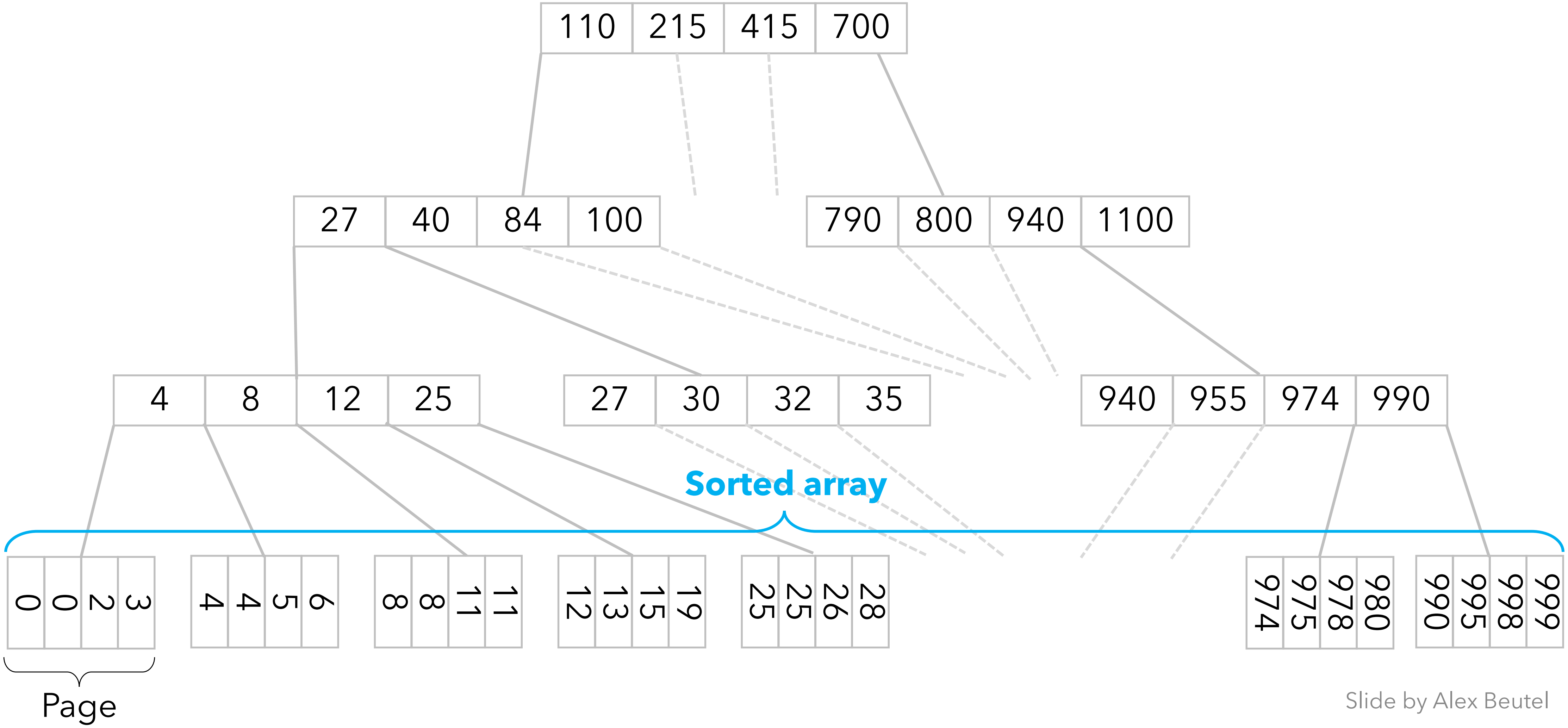
Outline

Goal: Use machine learning to augment

1. **Range index structures** (B-trees)
2. Existence index structures (Bloom filters)

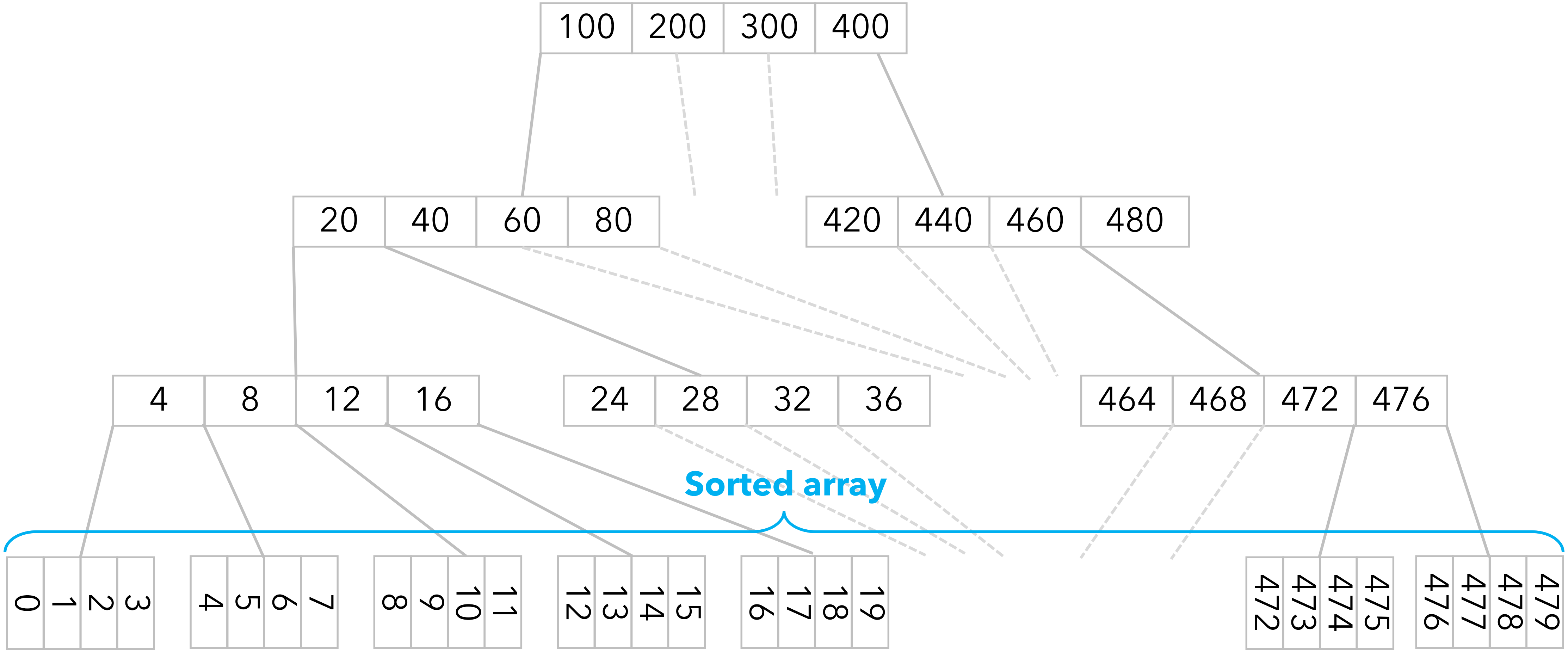
Kraska, Beutel, Chi, Dean, Polyzotis [SIGMOD'18]

B-trees



Slide by Alex Beutel

If data is all integers from 0 to 1 million?

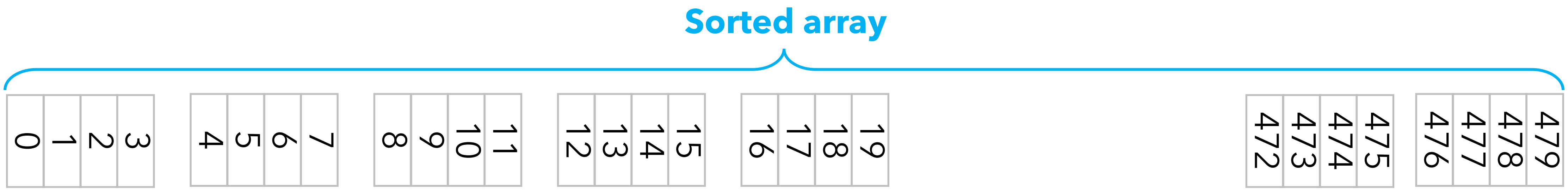


Slide by Alex Beutel

If data is all integers from 0 to 1 million?

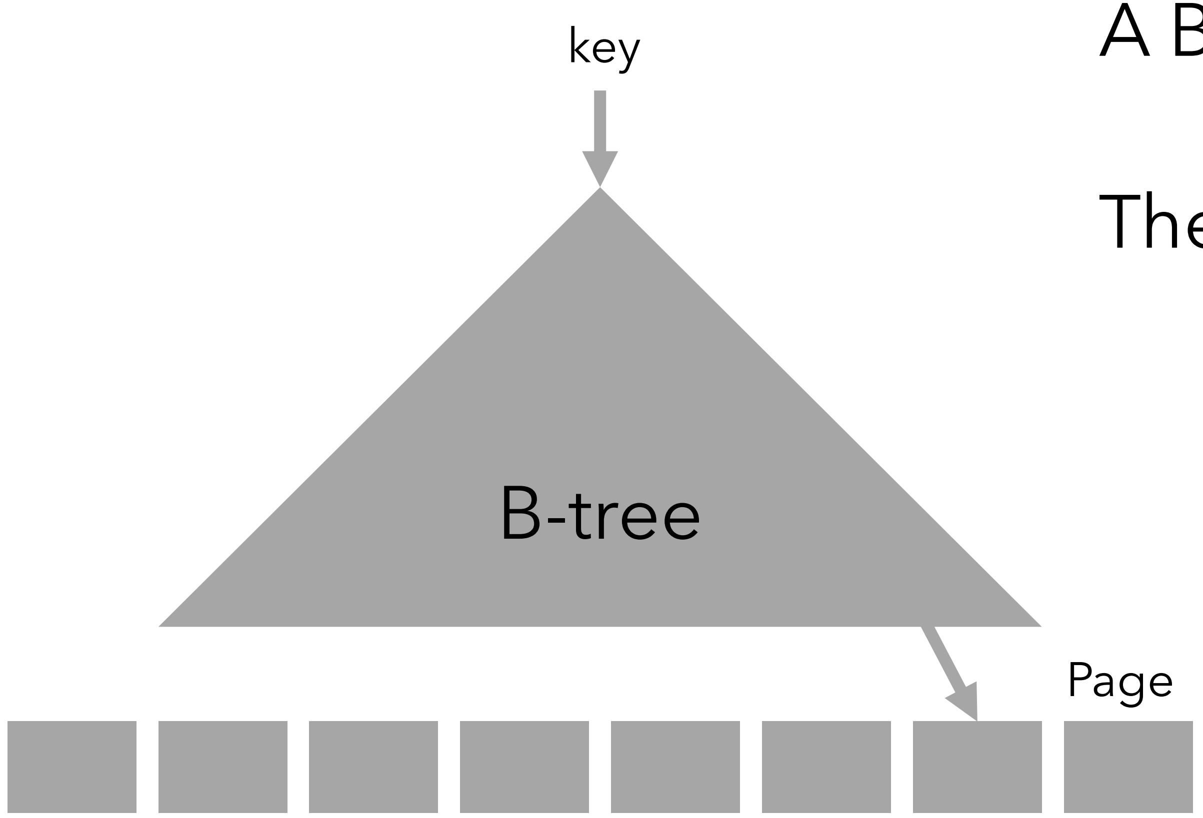
No need for B-tree

- O(1) look-up
- O(1) memory



Slide by Alex Beutel

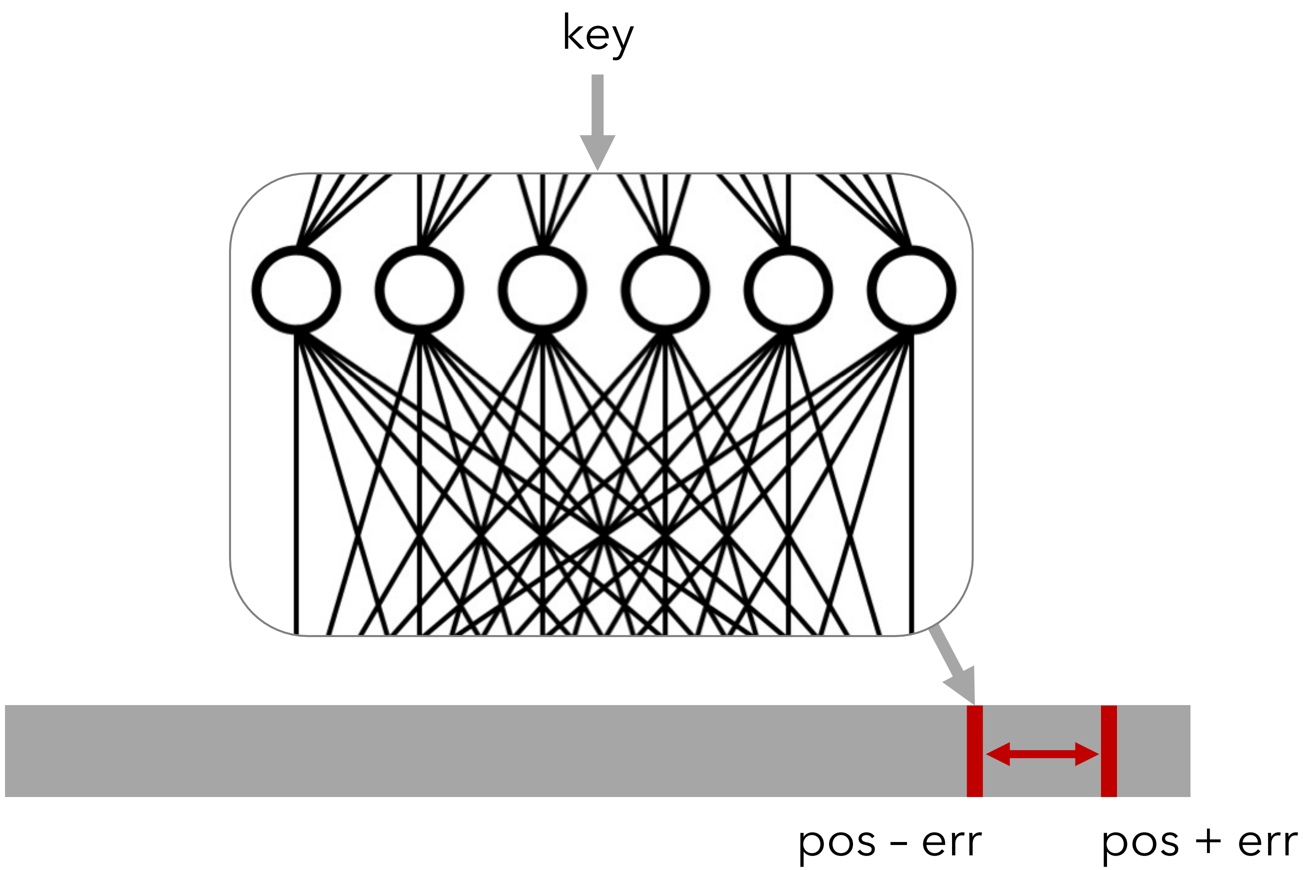
B-trees



A B-tree maps a key to a page

Then searches within the page

First attempt



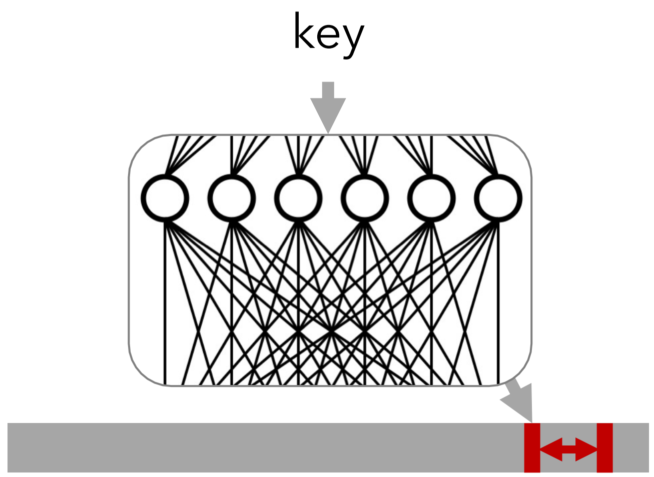
Replace B-tree
with **neural network**?

Model: f(key) → pos

Then searches from
[pos – err, pos + err]

Slide by Alex Beutel

First attempt



200M serve logs timestamp sorted

2-layer NN, 32-width fully-connected, ReLU

Tensorflow

B-trees lookup time: 300ns

Model lookup time: 80,000ns

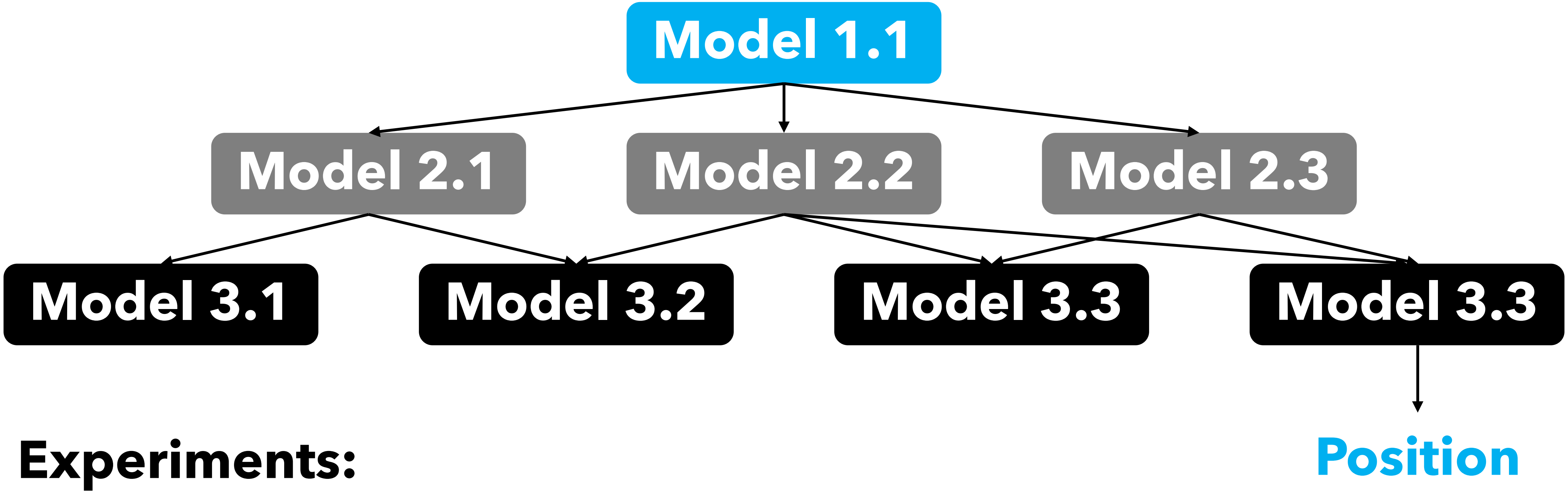
First attempt: key issues

1. Tensorflow designed for **big models**
   Big **overhead** on small models

2. B-Trees **"overfit"** the data…
   but NNs designed for **generalization**

3. B-trees are **cache-** and **operation-efficient**

Revised approach: Hierarchy of experts



Experiments:

- Up to 70% speed optimization
- Order-of-magnitude memory savings

Outline

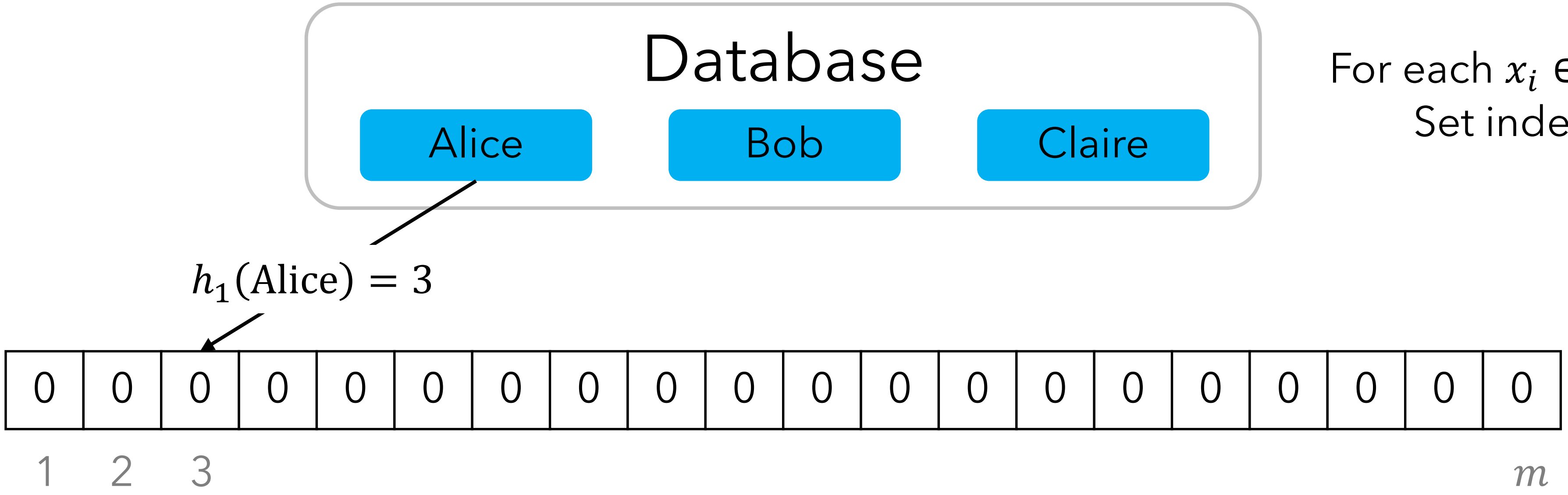
Goal: Use machine learning to augment

1. Range index structures (B-trees)
2. **Existence index structures** (Bloom filters)

Kraska, Beutel, Chi, Dean, Polyzotis [SIGMOD'18]

Bloom filters

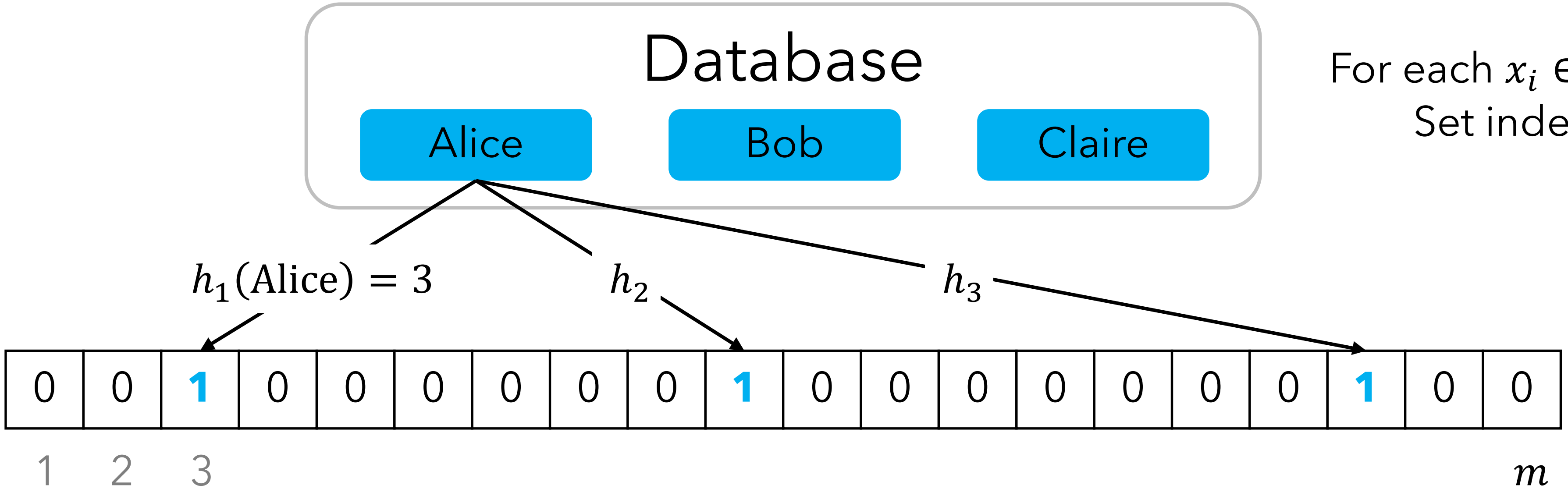
- Database is a set $K \subseteq U$
- **Goal:** DS allowing us to quickly determine if any $x \in U$ is in K
  - Use hash functions $h_1, h_2, \dots, h_k : U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$



For each $x_i \in K$ and $j \in [k]$:
Set index $h_j(x_i)$ equal to 1

Bloom filters

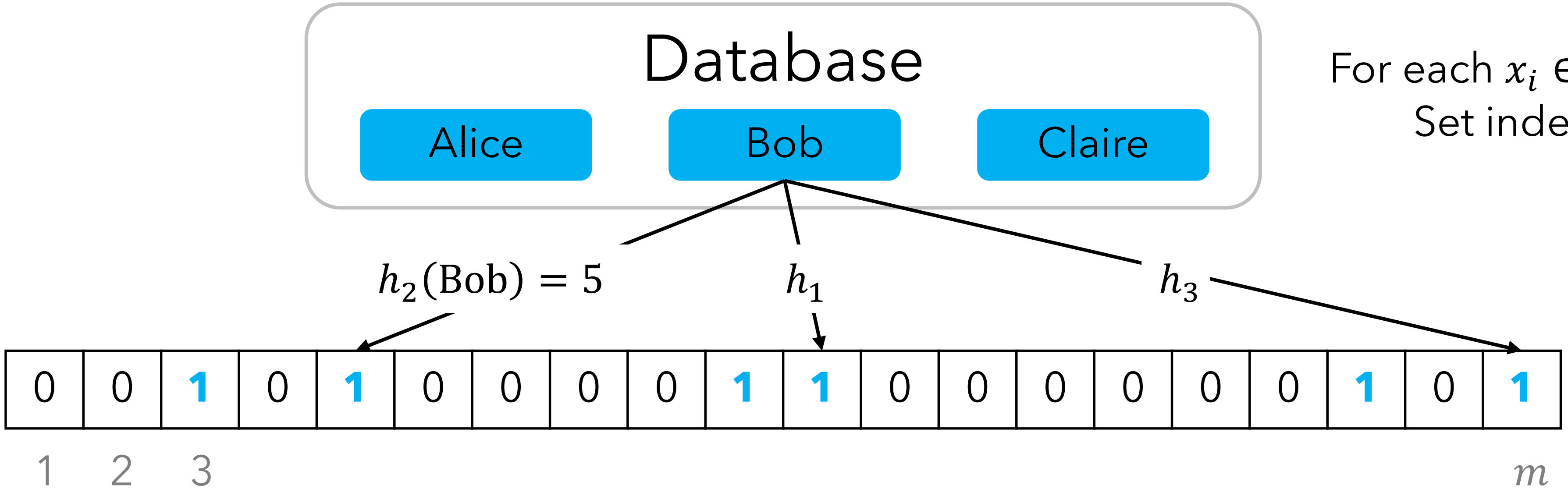
- Database is a set $K \subseteq U$
- **Goal:** DS allowing us to quickly determine if any $x \in U$ is in K
  - Use hash functions $h_1, h_2, \dots, h_k : U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$



For each $x_i \in K$ and $j \in [k]$:
Set index $h_j(x_i)$ equal to 1

Bloom filters

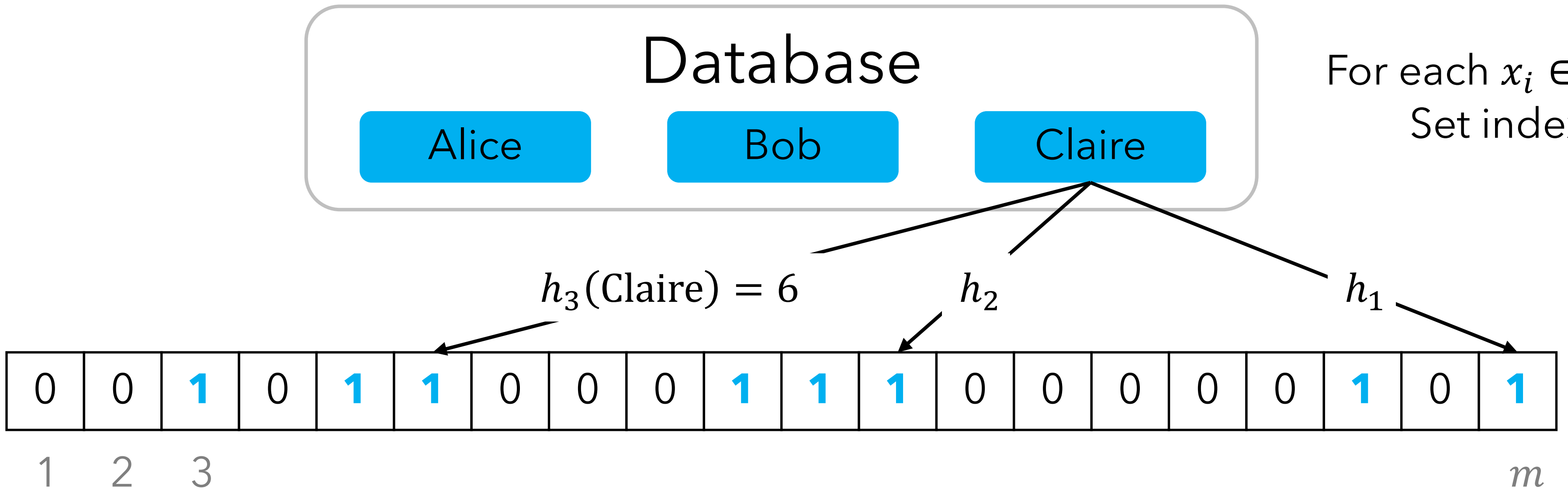
- Database is a set $K \subseteq U$
- **Goal:** DS allowing us to quickly determine if any $x \in U$ is in K
  - Use hash functions $h_1, h_2, \dots, h_k : U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$



For each $x_i \in K$ and $j \in [k]$:
Set index $h_j(x_i)$ equal to 1

Bloom filters

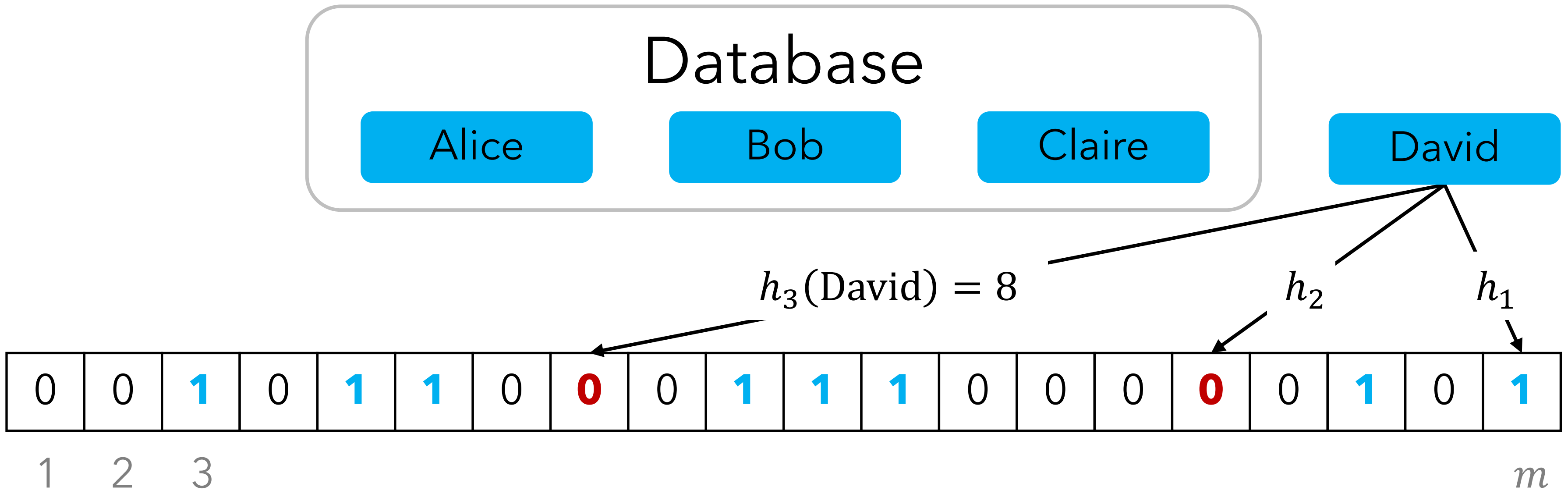
- Database is a set $K \subseteq U$
- **Goal:** DS allowing us to quickly determine if any $x \in U$ is in K
  - Use hash functions $h_1, h_2, \dots, h_k : U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$



For each $x_i \in K$ and $j \in [k]$:
Set index $h_j(x_i)$ equal to 1

Bloom filters

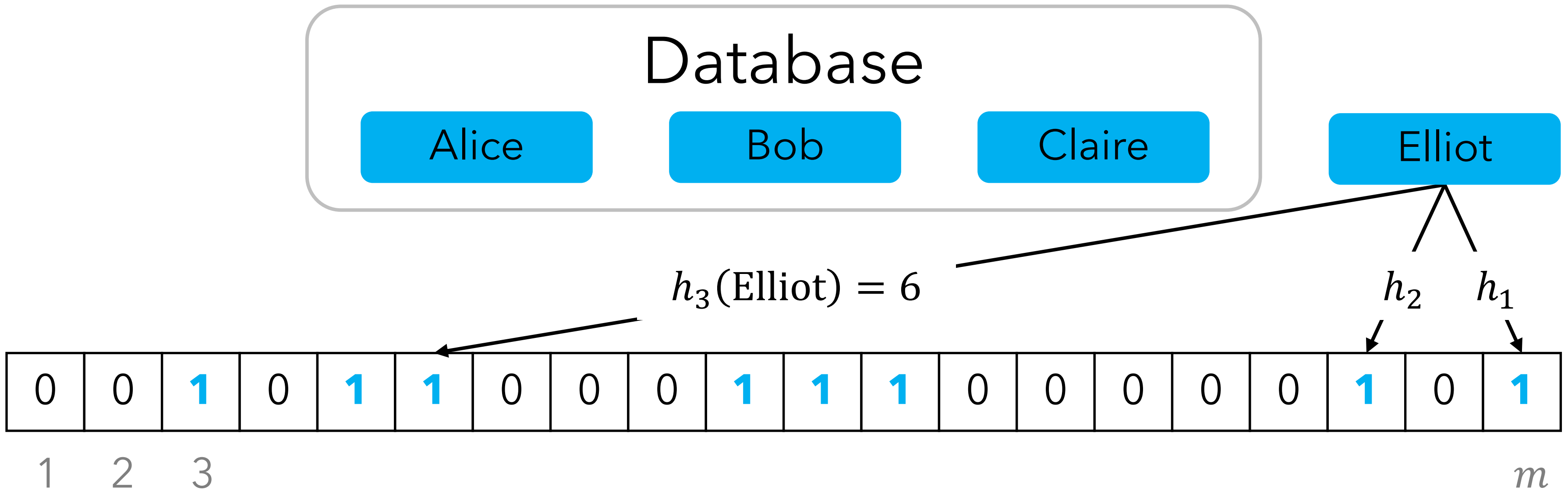
- Database is a set $K \subseteq U$
- **Goal:** DS allowing us to quickly determine if any $x \in U$ is in K
  - Use hash functions $h_1, h_2, \ldots, h_k : U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$



Not in database

Bloom filters

- Database is a set $K \subseteq U$
- **Goal:** DS allowing us to quickly determine if any $x \in U$ is in K
  - Use hash functions $h_1, h_2, \ldots, h_k : U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$



False positive
(no false negatives)

Learned Bloom filters

Idea 1: Replace Bloom filter with a classifier?

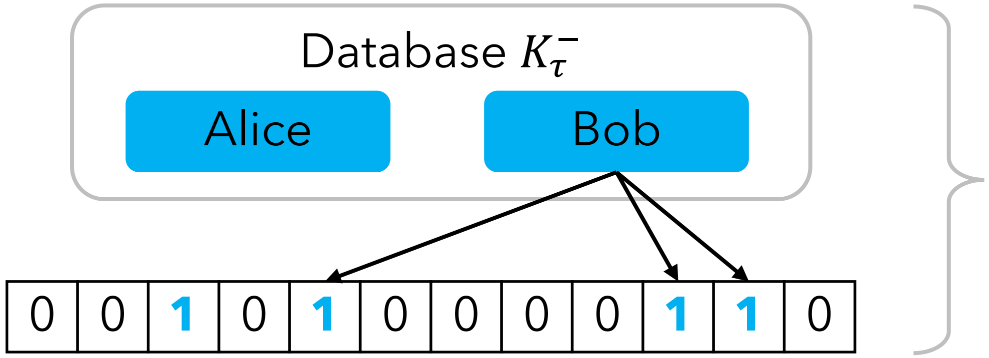
- Train ML model $f: U \to [0,1]$ with threshold τ so (hopefully)
$$f(x) \geq \tau \quad \Leftrightarrow \quad x \in K$$
- Training set: $(x, 1)$ for some $x \in K$, $(x, 0)$ for some $x \notin K$

Key issue: May be false negatives ($f(x) < \tau$ for some $x \in K$)

Learned Bloom filters

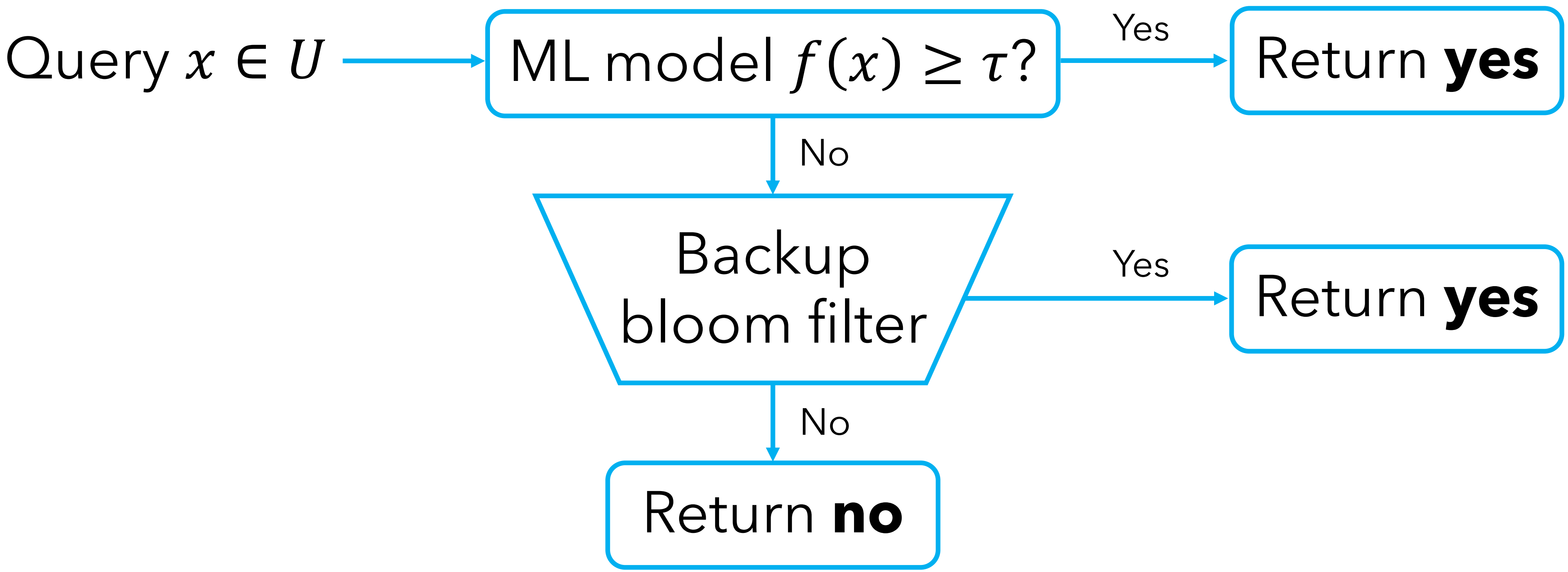
Idea 2:

- Train ML model $f: U \to [0,1]$ with threshold τ so (hopefully)
$$f(x) \geq \tau \quad \Leftrightarrow \quad x \in K$$
- Training set: $(x, 1)$ for some $x \in K$, $(x, 0)$ for some $x \notin K$
- Construct **backup Bloom filter:**
  - Bloom filter for set $K_\tau^- = \{x \in K : f(x) < \tau\}$



Ideally *much smaller* than Bloom filter for K

Learned Bloom filters



Setting τ

- D is a distribution over $U \setminus K$
- FPR_B = false positive rate of **backup Bloom filter**
  Next class: how to set Bloom filter size to control FPR_B
- False positive rate of **learned Bloom filter**:

$$\text{FPR}_O = \underbrace{\mathbb{P}_{x\sim D}[f(x) > \tau]}_{\text{Return } \textbf{yes}} + \underbrace{\mathbb{P}_{x\sim D}[f(x) \le \tau]}_{\text{Send to backup Bloom filter}} \cdot \text{FPR}_B$$

- If want $\text{FPR}_O \le \epsilon$:
  - Choose τ s.t. $\mathbb{P}_{x\sim D}[f(x) > \tau] \le \frac{\epsilon}{2}$
  - $\text{FPR}_B \le \frac{\epsilon}{2}$

Experiments

Keys: 1.7M URLs from a Google dataset

Non-keys: Random URLs and phishing URLs

Goal: 1% FPR

- Normal Bloom filter: 2.04MB
- Learned Bloom filter:
  - ML model (RNN) requires 0.0259MB
  - Backup Bloom filter requires 1.31MB

36% improvement

Outline

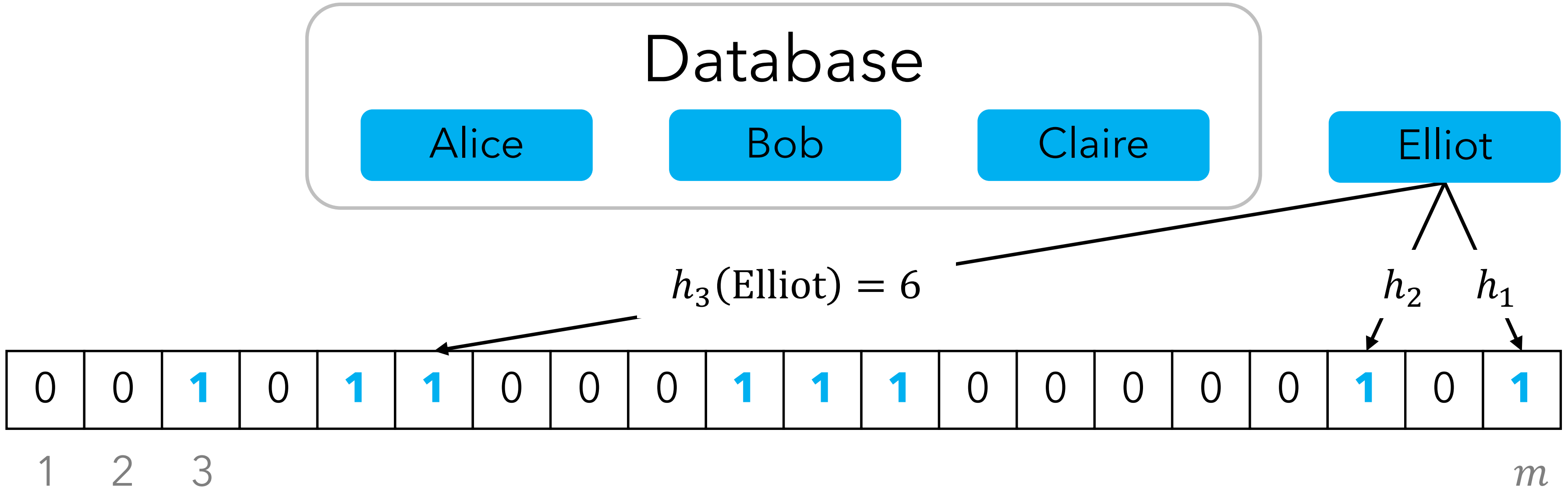
Goal: Use machine learning to augment

1. Range index structures (B-trees)
2. Existence index structures (Bloom filters)
   i. Approach
   ii. Theoretical guarantees/insights

Broder, Mitzenmacher [Internet Mathematics '04]
Mitzenmacher [NeurIPS'18]

Bloom filters

- Database is a set $K \subseteq U$
- Hash functions $h_1, h_2, \ldots, h_k : U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$



False positive
(no false negatives)

False positive analysis

- Database is a set $K \subseteq U$
- Hash functions $h_1, h_2, \ldots, h_k: U \to [m]$
  - Map $x \in U$ to a random number in $[m]$, independently and uniformly
- Bloom filter: array $A \in \{0,1\}^m$
- ρ = fraction of bits set to 1
- For any $y \notin K$,

$$\begin{aligned} & \mathbb{P}[y \text{ yields a false positive} \mid \rho = q] \\ = {} & \mathbb{P}[A[h_i(y)] = 1, \forall i \in [m] \mid \rho = q] \\ = {} & \mathbb{P}[A[h_1(y)] = 1 \mid \rho = q] \cdots \mathbb{P}[A[h_k(y)] = 1 \mid \rho = q] \\ = {} & q^k \end{aligned}$$

False positive analysis

- Database is a set $K \subseteq U$
- Hash functions $h_1, h_2, \dots, h_k : U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$
- ρ = fraction of bits set to 1

$$\mathbb{E}[\rho] = \mathbb{E}\left[\frac{1}{m}\sum_{i=1}^{m} \mathbf{1}_{\{i^{\text{th}} \text{ bit set to } 1\}}\right] = \frac{1}{m}\sum_{i=1}^{m} \mathbb{P}\left[i^{\text{th}} \text{ bit set to } 1\right]$$

False positive analysis

- Database is a set $K \subseteq U$
- Hash functions $h_1, h_2, \dots, h_k: U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$
- ρ = fraction of bits set to 1

$$\mathbb{E}[\rho] = \mathbb{E}\left[\frac{1}{m}\sum_{i=1}^{m} \mathbf{1}_{\{i^{\text{th}}\text{ bit set to 1}\}}\right] = \frac{1}{m}\sum_{i=1}^{m} \mathbb{P}\left[i^{\text{th}}\text{ bit set to 1}\right]$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left(1 - \mathbb{P}\left[i^{\text{th}}\text{ bit set to 0}\right]\right)$$

False positive analysis

- Database is a set $K \subseteq U$
- Hash functions $h_1, h_2, \ldots, h_k : U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$
- ρ = fraction of bits set to 1

$$\mathbb{E}[\rho] = \frac{1}{m} \sum_{i=1}^{m} \left(1 - \mathbb{P}\left[i^{\text{th}} \text{ bit set to } 0\right]\right)$$

$$\mathbb{P}\left[i^{\text{th}} \text{ bit set to } 0\right] = \mathbb{P}\left[h_j(x) \neq i, \forall x \in K, \forall j \in [k]\right]$$

False positive analysis

- Database is a set $K \subseteq U$
- Hash functions $h_1, h_2, \dots, h_k : U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$
- ρ = fraction of bits set to 1

$$\mathbb{E}[\rho] = \frac{1}{m}\sum_{i=1}^{m}\left(1 - \mathbb{P}\left[i^{\text{th}} \text{ bit set to } 0\right]\right)$$

$$\mathbb{P}\left[i^{\text{th}} \text{ bit set to } 0\right] = \mathbb{P}\left[h_j(x) \neq i, \forall x \in K, \forall j \in [k]\right]$$

$$\mathbb{P}\left[h_j(x) \neq i\right] = 1 - \mathbb{P}\left[h_j(x) = i\right] = 1 - \frac{1}{m}$$

False positive analysis

- Database is a set $K \subseteq U$
- Hash functions $h_1, h_2, \ldots, h_k: U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$
- ρ = fraction of bits set to 1

$$\mathbb{E}[\rho] = \frac{1}{m} \sum_{i=1}^{m} \left(1 - \mathbb{P}\left[i^{\text{th}} \text{ bit set to } 0\right]\right)$$

$$\mathbb{P}\left[i^{\text{th}} \text{ bit set to } 0\right] = \mathbb{P}\left[h_j(x) \neq i, \forall x \in K, \forall j \in [k]\right]$$

$$= \left(1 - \frac{1}{m}\right)^{|K|k}$$

False positive analysis

- Database is a set $K \subseteq U$
- Hash functions $h_1, h_2, \ldots, h_k \colon U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$
- ρ = fraction of bits set to 1

$$\mathbb{E}[\rho] = \frac{1}{m}\sum_{i=1}^{m}\left(1 - \mathbb{P}\left[i^{\text{th}} \text{ bit set to } 0\right]\right) = \frac{1}{m}\sum_{i=1}^{m}\left(1 - \left(1 - \frac{1}{m}\right)^{|K|k}\right)$$

$$\approx 1 - \exp\left(-\frac{|K|k}{m}\right)$$

- With high probability, $\rho \approx \mathbb{E}[\rho]$ (Chernoff bound)

False positive analysis

- Database is a set $K \subseteq U$
- Hash functions $h_1, h_2, \dots, h_k : U \to [m]$; $\mathbb{P}[h_i(x) = j] = \frac{1}{m}$
- ρ = fraction of bits set to 1
- For any $y \notin K$,

$$\mathbb{P}[y \text{ yields a false positive}] \approx \mathbb{E}[\rho]^k \approx \left(1 - \exp\left(-\frac{|K|k}{m}\right)\right)^k$$

  - If $m \approx |K| \log\frac{1}{\epsilon}$ and $k = \log\frac{1}{\epsilon}$,

$$\mathbb{P}[y \text{ yields a false positive}] \approx \epsilon$$

Differences between FPRs

Bloom filter: **for any** $y \notin K$, $\mathbb{P}[y \text{ yields a false positive}] \leq \epsilon$

Learned Bloom filter:

- D is a distribution over $U \setminus K$
- FPR_B = false positive rate of **backup Bloom filter**
- False positive rate of **learned Bloom filter**:

$$\mathrm{FPR}_O = \underbrace{\mathbb{P}_{y \sim D}[f(y) > \tau]}_{\text{Return } \textbf{yes}} + \underbrace{\mathbb{P}_{y \sim D}[f(y) \leq \tau]}_{\text{Send to backup Bloom filter}} \cdot \mathrm{FPR}_B$$

- FPR_O is with respect to a **random draw** of $y \sim D$

Differences between FPRs

Bloom filter: for any $y \notin$ … tive] $\leq \epsilon$

Robust **to distribution shift**

Learned Bloom filter:

Not robust **to distribution shift**

- D is a distribution over $U \setminus K$
- FPR_B = false positive rate of **backup Bloom filter**
- False positive rate of **learned Bloom filter**:

$$\mathrm{FPR}_O = \underbrace{\mathbb{P}_{y\sim D}[f(y) > \tau]}_{\text{Return yes}} + \underbrace{\mathbb{P}_{y\sim D}[f(y) \leq \tau]}_{\text{Send to backup Bloom filter}} \cdot \mathrm{FPR}_B$$

- FPR_O is with respect to a **random draw** of $y \sim D$